Relatively large θ_{13} and nearly maximal θ_{23} from the approximate S_3 symmetry of lepton mass matrices

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Abstract

We apply the permutation symmetry S_3 to both charged-lepton and neutrino mass matrices, and suggest a useful symmetry-breaking scheme, in which the flavor symmetry is explicitly broken down via $S_3 \to Z_3 \to \emptyset$ in the charged-lepton sector and via $S_3 \to Z_2 \to \emptyset$ in the neutrino sector. Such a two-stage breaking scenario is reasonable in the sense that both Z_3 and Z_2 are the subgroups of S_3, while Z_3 and Z_2 only have a trivial subgroup. In this scenario, we can obtain a relatively large value of the smallest neutrino mixing angle, e.g., $\theta_{13} \approx 9°$, which is compatible with the recent result from T2K experiment and will be precisely measured in the ongoing Double Chooz and Daya Bay reactor neutrino experiments. Moreover, the maximal atmospheric mixing angle $\theta_{23} \approx 45°$ can also be obtained while the best-fit value of solar mixing angle $\theta_{12} \approx 34°$ is assumed, which cannot be achieved in previous S_3 symmetry models.

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I.   INTRODUCTION

Flavor symmetry is currently a promising and widely-adopted approach to understanding lepton mass spectra and neutrino mixing pattern [1]. In particular, a lot of attention has recently been paid to discrete flavor symmetries, such as A_4 [2–5] and S_4 [6–14], which are able to predict the tri-bimaximal neutrino mixing with $\theta_{12}^{\rm T} = 35.3°$, $\theta_{23}^{\rm T} = 45°$ and $\theta_{13}^{\rm T} = 0$ that is well compatible with neutrino oscillation experiments [15–18]. However, the latest result from T2K experiment indicates that θ_{13} is likely to be not vanishing but relatively large. At the 90% confidence level, the T2K data are consistent with

$$5.0° \lesssim \theta_{13} \lesssim 16.0° \; , \tag{1}$$

in the case of normal neutrino mass hierarchy; and

$$5.8° \lesssim \theta_{13} \lesssim 17.8° \; , \tag{2}$$

in the case of inverted neutrino mass hierarchy, for a vanishing Dirac CP-violating phase $\delta = 0$ [19]. As a matter of fact, the global-fit analyses of neutrino oscillation experiments before the T2K result have already shown some hint on a nonzero θ_{13} [20–22]. For instance, the latest best-fit values of three neutrino mixing angles are $\theta_{12} = 34°$, $\theta_{23} = 46°$ and $\theta_{13} = 6°$ [22]. No doubt the symmetry-breaking terms in the A_4 or S_4 models can account for a relatively large θ_{13}, but one has to avoid the resultant large corrections to $\theta_{12}^{\rm T}$ and $\theta_{23}^{\rm T}$, which are already in excellent agreement with experimental data [23–26].

Therefore, we are well motivated to consider the simplest non-Abelian discrete symmetry S_3 for lepton mass matrices [27–46]. A salient feature of the S_3 model is the prediction of democratic neutrino mixing pattern [27, 28] with $\theta_{12}^{\rm D} = 45°$, $\theta_{23}^{\rm D} = 54.7°$ and $\theta_{13}^{\rm D} = 0$, which is now disfavored by current neutrino oscillation data. As argued in Refs. [47, 48], however, significant corrections from the symmetry-breaking terms may modify $\theta_{12}^{\rm D}$ and $\theta_{23}^{\rm D}$ to be consistent with the observed values, and simultaneously give rise to a relatively large θ_{13}. This observation is indeed intriguing because the perturbations to the symmetry-limit values of three mixing angles are comparable in magnitude.

In this paper, we reconsider the S_3 symmetry and its explicit breaking for lepton mass matrices, and demonstrate that a relatively large θ_{13} can be achieved while both θ_{12} and θ_{23} are in good agreement with neutrino oscillation experiments. Note that we shall follow

2

a phenomenological approach and work at the mass-matrix level, however, the derived patterns of lepton mass matrices and the proposed symmetry-breaking scheme may be helpful for the model building at the field-theory level. Our work differs from previous ones in several aspects. First, we apply the S_3 symmetry to both charged-lepton and neutrino mass matrices. In Refs. [41, 42, 45], the S_3 symmetry has only been applied to the neutrino mass matrix, and the exactly or nearly tri-bimaximal neutrino mixing can be derived. Second, we propose an interesting symmetry-breaking scheme, i.e., $S_3 \rightarrow Z_3 \rightarrow \emptyset$ for charged leptons and $S_3 \rightarrow Z_2 \rightarrow \emptyset$ for neutrinos. Such a two-stage breaking scenario is quite natural, because both Z_3 and Z_2 are the subgroups of S_3, while Z_3 and Z_2 only have a trivial subgroup. As a consequence of this breaking scheme, the charged-lepton mass matrix is non-symmetric, while neutrino mass matrix is still symmetric as it should be, because neutrinos are assumed to be Majorana particles. Third, we can get both a relatively large θ_{13} and a nearly maximal θ_{23}, which cannot be reached in the previous S_3 models [37, 44].

In Sec. II, the lepton mass matrices in the S_3-symmetry limit and the symmetry-breaking terms are constructed. The phenomenological implications for lepton mass spectra and neutrino mixing angles are explored in Sec. III. We summarize our conclusions in Sec. IV.

II. S_3 SYMMETRY AND ITS BREAKING

From the phenomenological point of view, the lepton masses and mixing angles at low energies are determined by lepton mass terms

$$-\mathcal{L}_\mathrm{m} = \overline{\ell_\mathrm{L}} M_\ell \ell_\mathrm{R} + \frac{1}{2}\overline{\nu_\mathrm{L}} M_\nu \nu_\mathrm{L}^c + \mathrm{h.c.} \ , \tag{3}$$

where M_ℓ stands for the mass matrix of charged leptons, and M_ν for the effective mass matrix of Majorana neutrinos. The latter can be realized in various neutrino mass models, such as seesaw models, which extend the standard model by introducing singlet or triplet fermions, or triplet scalars [49].

As usual, we can decompose the lepton mass matrices into a symmetry-limit part and a symmetry-breaking perturbation term:

$$M_l = M_l^{(0)} + \Delta M_l \ , \qquad M_\nu = M_\nu^{(0)} + \Delta M_\nu \ . \tag{4}$$

In the S_3-symmetry limit, the Lagrangian in Eq. (3) is invariant under the transformation $\ell_\mathrm{L} \rightarrow S^{(ijk)}\ell_\mathrm{L}$, $\ell_\mathrm{R} \rightarrow S^{(ijk)}\ell_\mathrm{R}$ and $\nu_\mathrm{L} \rightarrow S^{(ijk)}\nu_\mathrm{L}$ with $S^{(ijk)}$ being the group elements of S_3.

The three-dimensional representations of all six group elements are

$$
S^{(123)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \qquad S^{(231)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} ,
$$

$$
S^{(312)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} , \qquad S^{(213)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} ,
$$

$$
S^{(132)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} , \qquad S^{(321)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} . \tag{5}
$$

Thus $M_\ell^{(0)}$ and $M_\nu^{(0)}$ should commute with $S^{(ijk)}$, i.e., $[M_\ell^{(0)}, S^{(ijk)}] = 0$ and $[M_\nu^{(0)}, S^{(ijk)}] = 0$. The most general form of $M_\ell^{(0)}$ and $M_\nu^{(0)}$ with S_3 symmetry is [44]

$$
M_\ell^{(0)} = \frac{c_\ell}{3} \left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + r_\ell \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] ,
$$

$$
M_\nu^{(0)} = c_\nu \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right] , \tag{6}
$$

where the real and positive parameters c_ℓ and c_ν set the mass scales of charged leptons and neutrinos, respectively. Since both $M_\ell^{(0)}$ and $M_\nu^{(0)}$ can be diagonalized by the same orthogonal matrix

$$
V_{\rm D} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix} , \tag{7}
$$

the neutrino mixing matrix turns out to be an identity matrix. In other words, the democratic mixing arising from the charged-lepton sector gets too large corrections from the neutrino sector, or vice versa. Additionally, the first two generations of leptons are exactly degenerate in mass. In the limit of $r_\nu = 0$, the neutrino mass matrix is diagonal and we obtain the democratic mixing. But neutrinos are exactly degenerate in mass. The pertur-

bation terms explicitly breaking the S_3 symmetry are necessary to generate realistic lepton mass spectra and neutrino mixing angles.

Note that the group elements of S_3 can be categorized into three conjugacy classes $\mathcal{C}_0 = \{S^{(123)}\}$, $\mathcal{C}_1 = \{S^{(231)}, S^{(312)}\}$ and $\mathcal{C}_2 = \{S^{(213)}, S^{(132)}, S^{(321)}\}$. It is straightforward to show that the invariant subgroup of S_3 is the cyclic group of order three

$$Z_3 = \{S^{(123)}, S^{(231)}, S^{(312)}\} \equiv \{e, a, a^2\} \ , \tag{8}$$

where we have defined the identity element as $e \equiv S^{(123)}$ and the generator of Z_3 group as $a \equiv S^{(231)}$. With the explicit representations in Eq. (5), one can immediately verify $a^3 = e$. The S_3 group has three Z_2 subgroups

$$\begin{aligned} Z_2^{(12)} &= \{S^{(123)}, S^{(213)}\} \ , \\ Z_2^{(23)} &= \{S^{(123)}, S^{(132)}\} \ , \\ Z_2^{(31)} &= \{S^{(123)}, S^{(321)}\} \ . \end{aligned} \tag{9}$$

The $Z_2^{(23)}$ group can be identified with the μ-τ symmetry, which has been extensively discussed in connection with the maximal atmospheric mixing angle and the small reactor mixing angle [50, 51].

Obviously, it is natural to explicitly break S_3 symmetry to its subgroups. Along this line, we propose to construct the perturbation terms ΔM_ℓ and ΔM_ν as

$$\begin{aligned} \Delta M_\ell &= \Delta M_\ell^{(1)} + \Delta M_\ell^{(2)} \ , \\ \Delta M_\nu &= \Delta M_\nu^{(1)} + \Delta M_\nu^{(2)} \ , \end{aligned} \tag{10}$$

such that the flavor symmetry is explicitly broken down via the chain

$$\begin{matrix} & \Delta M_\ell^{(1)} & & \Delta M_\ell^{(2)} & \\ S_3 & \longrightarrow & Z_3 & \longrightarrow & \emptyset \end{matrix} \tag{11}$$

in the charged-lepton sector, while via a distinct chain

$$\begin{matrix} & \Delta M_\nu^{(1)} & & \Delta M_\nu^{(2)} & \\ S_3 & \longrightarrow & Z_2 & \longrightarrow & \emptyset \end{matrix} \tag{12}$$

in the neutrino sector. It is worthwhile to remark that the mass term breaking S_3 to Z_3 is proportional to $S^{(231)}$ or $S^{(312)}$, which is a non-symmetric matrix, thus the symmetry-breaking chain in Eq. (11) is only allowed for charged leptons. Neutrino mass matrix must

be symmetric because we have assumed neutrinos to be Majorana particles as in a class of seesaw models.

It is easy to show that $\Delta M_\ell^{(1)}$ can always be cast into the following form

$$\Delta M_\ell^{(1)} = \frac{c_\ell}{3} \begin{pmatrix} 0 & \delta_\ell & 0 \\ 0 & 0 & \delta_\ell \\ \delta_\ell & 0 & 0 \end{pmatrix} \tag{13}$$

by redefining the parameters c_ℓ and r_ℓ in $M_\ell^{(0)}$. The first-order perturbation term in the neutrino mass matrix can be written as

$$\Delta M_\nu^{(1)} = c_\nu \begin{pmatrix} \delta_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \\ 0 & \delta_\nu & 0 \end{pmatrix} , \tag{14}$$

which reduces the S_3 symmetry to $Z_2^{(23)}$ or μ-τ symmetry. Note that here we take $Z_2^{(23)}$ for example, and one can discuss similarly the other two possibilities $Z_2^{(12)}$ and $Z_2^{(31)}$. Nevertheless, the second-stage perturbation terms $\Delta M_\ell^{(2)}$ and $\Delta M_\nu^{(2)}$, which respectively break down the residual Z_3 and Z_2 symmetries, could have many different forms. As both of them are intended for breaking the mass degeneracy between the first and second generations, we choose the diagonal form for simplicity [37]

$$\Delta M_\ell^{(2)} = \frac{c_\ell}{3} \begin{pmatrix} -i\epsilon_\ell & 0 & 0 \\ 0 & +i\epsilon_\ell & 0 \\ 0 & 0 & +\varepsilon_\ell \end{pmatrix} ,$$

$$\Delta M_\nu^{(2)} = c_\nu \begin{pmatrix} -\epsilon_\nu & 0 & 0 \\ 0 & +\epsilon_\nu & 0 \\ 0 & 0 & +\varepsilon_\nu \end{pmatrix} . \tag{15}$$

Now that $\Delta M_\ell^{(2)}$ is complex, we expect the CP violation in the lepton sector. Furthermore, all the parameters r_f, δ_f, ϵ_f and ε_f for $f = \ell, \nu$ are assumed to be real and serve as small perturbations, i.e., $|r_f|, |\delta_f|, |\epsilon_f| \ll |\varepsilon_f| < 1$. At this moment, we have completed the construction of the lepton mass matrices in Eq. (4).

III.   LEPTON MASSES AND MIXING ANGLES

Now we are ready to figure out the lepton mass spectra and neutrino mixing angles. In general, the charged-lepton mass matrix M_ℓ is an arbitrary complex matrix and can be diagonalized by a bi-unitary transformation $U_\ell^\dagger M_\ell \tilde{U}_\ell = \text{Diag}\{m_e, m_\mu, m_\tau\}$, where m_α (for $\alpha = e, \mu, \tau$) are charged-lepton masses and the matrices U_ℓ and $\tilde{U}_\ell$ are unitary. Since $\tilde{U}_\ell$ is associated with the right-handed fields of charged leptons and has nothing to do with the lepton flavor mixing, it is more convenient to consider the Hermitian matrix $H_\ell \equiv M_\ell M_\ell^\dagger$, which can be diagonalized as $U_\ell^\dagger H_\ell U_\ell = \text{Diag}\{m_e^2, m_\mu^2, m_\tau^2\}$. Furthermore, we shall work in the so-called hierarchy basis, where the relevant matrix is $H_\ell' \equiv V_\text{D}^T H_\ell V_\text{D}$. In the leading-order approximation, we arrive at

$$H_\ell' = \frac{c_\ell^2}{9} \begin{pmatrix} r_\ell^2 - r_\ell \delta_\ell + \delta_\ell^2 + \epsilon_\ell^2 & \dfrac{\delta_\ell \varepsilon_\ell}{\sqrt{3}} & -i\sqrt{6}\epsilon_\ell \\ \dfrac{\delta_\ell \varepsilon_\ell}{\sqrt{3}} & \dfrac{2}{3}\varepsilon_\ell(\varepsilon_\ell + 2r_\ell - \delta_\ell) & -\sqrt{2}\varepsilon_\ell \\ i\sqrt{6}\epsilon_\ell & -\sqrt{2}\varepsilon_\ell & 9 + 2\varepsilon_\ell \end{pmatrix} \tag{16}$$

with a rational assumption of $|r_\ell|, |\delta_\ell|, |\epsilon_\ell| \ll |\varepsilon_\ell| < 1$. After diagonalizing the above matrix via $V_\ell^\dagger H' V_\ell = \text{Diag}\{m_e^2, m_\mu^2, m_\tau^2\}$, one obtains three charged-lepton masses

$$\begin{aligned} m_e &\approx c_\ell \left| \frac{r_\ell}{3} - \frac{\delta_\ell}{6} + \frac{\epsilon_\ell^2}{6\varepsilon_\ell} + \frac{3\delta_\ell^2}{8\varepsilon_\ell} \right| , \\ m_\mu &\approx c_\ell \left(\frac{2}{9}\varepsilon_\ell + \frac{1}{3}r_\ell - \frac{1}{6}\delta_\ell \right) , \\ m_\tau &\approx c_\ell \left(1 + \frac{1}{9}\varepsilon_\ell + \frac{1}{3}r_\ell + \frac{1}{3}\delta_\ell \right) . \end{aligned} \tag{17}$$

Defining $m_0 = c_\ell(2r_\ell - \delta_\ell)/6$, we have $|m_0| < m_\mu$. The small parameters ε_ℓ, δ_ℓ and ϵ_ℓ can be expressed in terms of charged-lepton masses and the m_0 parameter

$$\varepsilon_\ell \approx \frac{9}{2}\frac{m_\mu - m_0}{m_\tau - m_0} , \qquad \frac{\epsilon_\ell^2}{\varepsilon_\ell^2} + \frac{9\delta_\ell^2}{4\varepsilon_\ell^2} \approx \frac{4}{3}\frac{|m_e - |m_0||}{m_\mu - m_0} . \tag{18}$$

For $\delta_\ell = 0$, one immediately reproduces the same results in Ref. [44], where the perturbation term $\Delta M_\ell^{(1)}$ is absent in the charged-lepton mass matrix. The unitary matrix $U_\ell = V_\text{D} V_\ell$ is found to be

$$U_\ell \approx V_D + \frac{e^{-i\phi}}{\sqrt{6}} \frac{\sqrt{|m_e - |m_0||}}{\sqrt{m_\mu - m_0}} \begin{pmatrix} 1 & \sqrt{3} & 0 \\ 1 & -\sqrt{3} & 0 \\ -2 & 0 & 0 \end{pmatrix} + \frac{1}{2\sqrt{3}} \frac{m_\mu - m_0}{m_\tau - m_0} \begin{pmatrix} 0 & \sqrt{2} & -1 \\ 0 & \sqrt{2} & -1 \\ 0 & \sqrt{2} & 2 \end{pmatrix}, \quad (19)$$

where $\phi \equiv \arctan[2\epsilon_\ell/(3\delta_\ell)]$ gives rise to the Dirac CP-violating phase. Comparing Eq. (19) with the counterpart in Ref. [44], we can observe that the additional symmetry-breaking term $\Delta M_\ell^{(1)}$ or the parameter δ_ℓ can be determined by measuring the CP violation in neutrino oscillations, which is indeed to be performed in the long-baseline neutrino experiments. It is worth mentioning that there are five real parameters in the charged-lepton mass matrix (i.e., $c_\ell, r_\ell, \delta_\ell, \epsilon_\ell$ and ε_ℓ), which can be expressed in terms of charged-lepton masses (m_e, m_μ, m_τ) and (m_0, ϕ). The latter two enter into the neutrino mixing matrix, and can be determined by neutrino mixing angles and the Dirac CP-violating phase, as we shall show later.

Next, we turn to the neutrino mass matrix given in Eqs. (6), (14) and (15). The unitary matrix U_ν used to diagonalize M_ν through $U_\nu^\dagger M_\nu U_\nu^* = \text{Diag}\{m_1, m_2, m_3\}$ is approximately given by [37, 44]

$$U_\nu \approx \frac{1}{\varepsilon_\nu} \begin{pmatrix} \varepsilon_\nu c_\theta & \varepsilon_\nu s_\theta & r_\nu \\ -\varepsilon_\nu s_\theta & \varepsilon_\nu c_\theta & r_\nu + \delta_\nu \\ (r_\nu + \delta_\nu)s_\theta - r_\nu c_\theta & -(r_\nu + \delta_\nu)c_\theta - r_\nu s_\theta & \varepsilon_\nu \end{pmatrix}, \quad (20)$$

where $c_\theta \equiv \cos\theta$ and $s_\theta = \sin\theta$ with $\tan 2\theta \equiv 2r_\nu/(2\epsilon_\nu - \delta_\nu)$. Note that the perturbation parameters satisfy $r_\nu, \delta_\nu, \epsilon_\nu \ll \varepsilon_\nu < 1$ as in the case of charged leptons. Three neutrino mass eigenvalues are

$$\begin{aligned} m_3 &\approx c_\nu \left(1 + r_\nu + \varepsilon_\nu\right), \\ m_2 &\approx c_\nu \left(1 + r_\nu + \delta_\nu/2 + \sqrt{(\epsilon_\nu - \delta_\nu/2)^2 + r_\nu^2}\right), \\ m_1 &\approx c_\nu \left(1 + r_\nu + \delta_\nu/2 - \sqrt{(\epsilon_\nu - \delta_\nu/2)^2 + r_\nu^2}\right), \end{aligned} \quad (21)$$

where we have assumed the normal mass hierarchy. It is straightforward to calculate the neutrino mass-square differences $\Delta m_{31}^2 \approx 2c_\nu^2 \varepsilon_\nu$ and $\Delta m_{21}^2 \approx 2c_\nu^2 \sqrt{(2\epsilon_\nu - \delta_\nu)^2 + 4r_\nu^2}$, for which the latest best-fit values are $\Delta m_{21}^2 = 7.59 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 = 2.45 \times 10^{-3}$ eV^2 [22]. As indicated by Eq. (21), we have nearly degenerate neutrino masses. Therefore, the effective neutrino mass in tritium beta decays $\langle m_\beta \rangle$ and that in neutrinoless double-beta

decays $\langle m_{\beta\beta}\rangle$ are on the same order of the mass-scale parameter c_ν. Currently, the most stringent bound $\langle m_\beta\rangle \approx \langle m_{\beta\beta}\rangle \approx c_\nu \sim \mathcal{O}(0.1\text{ eV})$ comes from cosmological observations [52]. With the help of neutrino mass-squared differences, we can estimate [37]

$$\varepsilon_\nu \approx \frac{\Delta m_{31}^2}{2\langle m_\beta\rangle^2} \approx 0.12 \,, \quad \sqrt{(2\epsilon_\nu - \delta_\nu)^2 + 4r_\nu^2} \approx \frac{\Delta m_{21}^2}{2\langle m_\beta\rangle^2} \approx 3.8 \times 10^{-3} \,. \tag{22}$$

In order to further fix the model parameters r_ν, δ_ν and ϵ_ν, we have to study neutrino mixing angles and the Dirac CP-violating phase.

From Eqs. (19) and (20), we can derive the neutrino mixing matrix, which is defined as $V \equiv U_\ell^\dagger U_\nu$. More explicitly,

$$
\begin{aligned}
V \approx\ & \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3}(c_\theta + s_\theta) & -\sqrt{3}(c_\theta - s_\theta) & 0 \\ (c_\theta - s_\theta) & (c_\theta + s_\theta) & -2 \\ \sqrt{2}(c_\theta - s_\theta) & \sqrt{2}(c_\theta + s_\theta) & \sqrt{2} \end{pmatrix} \\
&+ \frac{1}{2\sqrt{3}} \frac{m_\mu - m_0}{m_\tau - m_0} \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2}(c_\theta - s_\theta) & \sqrt{2}(c_\theta + s_\theta) & \sqrt{2} \\ -(c_\theta - s_\theta) & -(c_\theta + s_\theta) & 2 \end{pmatrix} \\
&+ \frac{e^{i\phi}}{\sqrt{6}} \frac{\sqrt{|m_e - |m_0||}}{\sqrt{m_\mu - m_0}} \begin{pmatrix} (c_\theta - s_\theta) & (c_\theta + s_\theta) & -2 \\ \sqrt{3}(c_\theta + s_\theta) & -\sqrt{3}(c_\theta - s_\theta) & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&+ \frac{1}{\sqrt{6}} \frac{r_\nu}{\epsilon_\nu} \begin{pmatrix} 0 & 0 & 0 \\ 2(c_\theta - s_\theta) & 2(c_\theta + s_\theta) & 2 \\ -\sqrt{2}(c_\theta - s_\theta) & -\sqrt{2}(c_\theta + s_\theta) & 2\sqrt{2} \end{pmatrix} + \frac{1}{\sqrt{6}} \frac{\delta_\nu}{\epsilon_\nu} \begin{pmatrix} 0 & 0 & -\sqrt{3} \\ -2s_\theta & 2c_\theta & 1 \\ \sqrt{2}s_\theta & -\sqrt{2}c_\theta & \sqrt{2} \end{pmatrix} \,,
\end{aligned} \tag{23}
$$

where the last term arises from the symmetry-breaking term $\Delta M_\nu^{(1)}$, which evidently contributes to both θ_{13} and θ_{23}. Comparing between Eq. (23) and the standard parametrization of neutrino mixing matrix [52], one can extract three neutrino mixing angles and the CP-violating phase. Some comments are in order:

1. The solar mixing angle θ_{12} is determined by $\sin^2 2\theta_{12} \approx 4|V_{e1}|^2|V_{e2}|^2 \approx \cos^2 2\theta$ with $\tan 2\theta = 2r_\nu/(2\epsilon_\nu - \delta_\nu)$, so the perturbation parameters r_ν, ϵ_ν and δ_ν should satisfy

$$\frac{2r_\nu}{|2\epsilon_\nu - \delta_\nu|} = \cot 2\theta_{12} = 0.4 \,, \tag{24}$$

9

where the best-fit value $\theta_{12} = 34°$ has been input [22]. Combining Eqs. (22) and (24), one can get $r_\nu \approx 7.0 \times 10^{-4} \ll \varepsilon_\nu$, which justifies our assumption $r_\nu, \delta_\nu, \epsilon_\nu \ll \varepsilon_\nu < 1$ for perturbation parameters. The ratio ϵ_ν/δ_ν is thus the only unfixed parameter in the neutrino sector.

2. The smallest neutrino mixing angle θ_{13} is given by

$$\sin\theta_{13} \approx \left| \frac{2}{\sqrt{6}} e^{i\phi} \frac{\sqrt{|m_e - |m_0||}}{\sqrt{m_\mu - m_0}} + \frac{1}{\sqrt{2}} \frac{\delta_\nu}{\varepsilon_\nu} \right| , \tag{25}$$

while the Dirac CP-violating phase by

$$\delta \approx \arg\left[\frac{2}{\sqrt{6}} e^{i\phi} \frac{\sqrt{|m_e - |m_0||}}{\sqrt{m_\mu - m_0}} + \frac{1}{\sqrt{2}} \frac{\delta_\nu}{\varepsilon_\nu} \right] . \tag{26}$$

Note that θ_{13} receives contributions both from charged-lepton and neutrino sectors. If $m_0 = 0$ and $\delta_\nu = 0$ are taken, as in Ref. [37], we get $\sin\theta_{13} \approx \sqrt{2m_e/3m_\mu} \approx 0.057$ or $\theta_{13} \approx 3.2°$ by inputting $m_e = 0.4866$ MeV and $m_\mu = 102.718$ MeV at the electroweak scale [53]. As observed in Ref. [44], when m_0 is switched on and set to $m_0 < 0$ and $|m_0| > m_e$, one can get relatively large values of θ_{13} and saturate the upper bound for $m_0 \approx -14\, m_e$. In our scenario, the sizable θ_{13} can be obtained even for somewhat smaller $|m_0|$ due to the $\delta_\nu/\varepsilon_\nu$ term.

3. The atmospheric mixing angle θ_{23} is given by

$$\sin 2\theta_{23} = \frac{2\sqrt{2}}{3}\left(1 + \frac{1}{2}\frac{m_\mu - m_0}{m_\tau - m_0} + \frac{r_\nu}{\varepsilon_\nu} + \frac{1}{2}\frac{\delta_\nu}{\varepsilon_\nu}\right) , \tag{27}$$

which can also be enhanced due to the symmetry-breaking term $\Delta M_\nu^{(1)}$ or the δ_ν parameter. If δ_ν is vanishing, one obtains $\sin 2\theta_{23} \approx 0.97$ or $\theta_{23} \approx 38°$ by inputting $m_\mu = 102.718$ MeV and $m_\tau = 1746.24$ MeV at the electroweak scale [53]. The nearly maximal mixing angle $\theta_{23} \approx 45°$ cannot be achieved even for $m_0 = -14\, m_e$, which is necessary to generate relatively-large θ_{13} [44]. As indicated by Eq. (27), however, θ_{23} can be nearly maximal in our scenario with a nonvanishing δ_ν.

To illustrate how our model can accommodate both relatively-large θ_{13} and nearly-maximal θ_{23}, we introduce $\xi \equiv \epsilon_\nu/\delta_\nu$ and $\zeta \equiv |m_0|/m_e$, and rewrite Eqs. (25) and (27) in terms of (ϕ, ξ, ζ) and physical observables

$$\sin\theta_{13} \approx \left| e^{i\phi}\sqrt{\frac{2|\zeta - 1|}{3(\zeta + m_\mu/m_e)}} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2}\frac{\sin 2\theta_{12}}{\sqrt{2}|2\xi - 1|} \right| , \tag{28}$$

and

$$\sin 2\theta_{23} \approx \frac{2\sqrt{2}}{3} \left[1 + \frac{1}{2} \frac{m_\mu/m_e + \zeta}{m_\tau/m_e + \zeta} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \left(\frac{\sin 2\theta_{12}}{2|2\xi - 1|} + \frac{\cos 2\theta_{12}}{2} \right) \right] , \qquad (29)$$

where $m_0 < 0$ and $\xi \neq 1/2$ have been assumed. Hence three remaining parameters (ϕ, ξ, ζ) are actually fixed by δ, θ_{13} and θ_{23}, which can be measured in neutrino oscillation experiments. Note that $|m_0| = |c_\ell(2r_\ell - \delta_\ell)|/6$ is naturally on the same order of m_e, thus $\sin 2\theta_{23}$ is insensitive to ζ because of the strong mass hierarchy of charged leptons $m_\tau \gg m_\mu \gg m_e$. In this case, we can safely neglect ζ in Eq. (29) and solve it analytically for the ξ parameter

$$\xi \approx \frac{1}{2} \pm \frac{\sin 2\theta_{12}}{4 \left[\left(\frac{3}{2\sqrt{2}} \sin 2\theta_{23} - \frac{1}{2} \frac{m_\mu}{m_\tau} - 1 \right) \frac{\Delta m_{31}^2}{\Delta m_{21}^2} - \frac{\cos 2\theta_{12}}{2} \right]} , \qquad (30)$$

where the upper and lower sign stands for $\xi > 1/2$ and $\xi < 1/2$, respectively. In assumption of $\phi = 0$, we can further solve Eq. (28) for the ζ parameter

$$\zeta \approx 1 + \frac{3m_\mu}{2m_e} \left(\sin\theta_{13} - \frac{3}{2} \sin 2\theta_{23} + \sqrt{2} + \frac{1}{\sqrt{2}} \frac{m_\mu}{m_\tau} - \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \frac{\cos 2\theta_{12}}{\sqrt{2}} \right)^2 . \qquad (31)$$

In order to obtain $\theta_{13} \approx 9°$ and $\theta_{23} \approx 45°$ as well, one can insert the best-fit values $\Delta m_{21}^2 = 7.59 \times 10^{-5}$ eV^2, $\Delta m_{31}^2 = 2.45 \times 10^{-3}$ eV^2 and $\theta_{12} = 34°$ [22], together with the charged-lepton masses, into Eqs. (30) and (31), and finally find $\zeta \approx 5$ and $\xi \approx 0.2$ or $\xi \approx 0.8$. In the neutrino sector, we can estimate the model parameters as $r_\nu \approx 7.0 \times 10^{-4}$, $\delta_\nu \approx 5.8 \times 10^{-3}$, $\epsilon_\nu \approx 1.16 \times 10^{-3}$ for $\xi = 0.2$ or $\epsilon_\nu \approx 4.64 \times 10^{-3}$ for $\xi = 0.8$, and $\varepsilon_\nu = 0.12$, which are consistent with the requirement that $r_\nu, \delta_\nu, \epsilon_\nu \ll \varepsilon_\nu$. In the charged-lepton sector, we get $m_0 \approx -5m_e$ from $\zeta \approx 5$, and have assumed $\phi = 0$. The latter condition implies $\epsilon_\ell = 0$, and thus one can find from Eq. (18) that $\varepsilon_\ell \approx 9m_\mu/(2m_\tau) \approx 0.26$, $\delta_\ell \approx 8\sqrt{m_e}\varepsilon_\ell/(3\sqrt{3m_\mu}) \approx 0.03$ and $r_\ell \approx \delta_\ell$, which are also in agreement with $r_\ell, \delta_\ell, \epsilon_\ell \ll \varepsilon_\ell$. Therefore, both $\theta_{13} \approx 9°$ and $\theta_{23} \approx 45°$ can indeed be achieved in our scenario. For the case with $\phi \neq 0$, we can completely determine all the model parameters, if the Dirac CP-violating phase δ is measured in the future neutrino oscillation experiments.

IV. REMARKS AND CONCLUSIONS

How to understand lepton mass spectra and neutrino mixing pattern remains an open question in elementary particle physics. Flavor symmetry is currently a powerful tool to tackle this longstanding problem. In this paper, we apply the S_3 symmetry to both charged-lepton and neutrino mass matrices. In order to explain realistic lepton mass spectra and

neutrino mixing angles, the S_3 symmetry is explicitly broken down via $S_3 \to Z_3 \to \emptyset$ in the charged-lepton sector, while via $S_3 \to Z_2 \to \emptyset$ in the neutrino sector. Along this line, the mass matrices of charged leptons and neutrinos are constructed step by step. Some interesting features of this model have been explored:

- It seems reasonable that the flavor symmetry first breaks down to its subgroups. The permutation group S_3 contains only two kinds of non-trivial subgroups, i.e., Z_3 and Z_2. For the breaking chain $S_3 \to Z_3 \to \emptyset$, the mass matrix has to be non-symmetric, so it is only allowed for charged leptons. Neutrinos are assumed to be Majorana particles, which are actually realized in various seesaw models. Therefore, neutrino mass matrix should be symmetric, which is not spoiled in the $S_3 \to Z_2 \to \emptyset$ breaking chain.

- After the flavor symmetry breaking, lepton mass matrices are determined by ten parameters, i.e., c_f, r_f, δ_f, ϵ_f and ε_f for $f = \ell, \nu$. It has been shown that all of them are completely fixed by the ten observables in the lepton sector, namely three charged-lepton masses (m_e, m_μ, m_τ), three neutrino masses (m_1, m_2, m_3), three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and one Dirac CP-violating phase δ. If leptonic CP violation is finally measured in the future long-baseline neutrino experiments, the model parameters will be fully determined. In light of the recent T2K indication of relatively-large θ_{13}, the discovery of CP violation in neutrino oscillations seems very promising.

- In our symmetry-breaking scheme, it has been found that θ_{13} and θ_{23} can receive large corrections from the S_3 symmetry-breaking terms. More explicitly, both corrections from charged leptons and neutrinos are significant for obtaining a relatively-large θ_{13}, while θ_{23} is mainly sensitive to the breaking term in the neutrino mass matrix. We show that both a relatively-large θ_{13} and a nearly-maximal θ_{23} can be accommodated simultaneously, which is not the case for previous S_3 symmetry models.

Finally, it is worth mentioning that the $S_3 \to Z_2 \to \emptyset$ chain can be applied to charged leptons. In addition, the two-stage breaking scheme could also be applicable to quarks, and may be helpful in understanding quark mass spectra, mixing angles and CP violation. It should be very interesting if the mass spectra and mixing patterns for both quarks and leptons can be understood in this way, and a renormalizable field-theory model with the S_3 symmetry can be constructed to realize the derived fermion mass matrices. We leave these issues for future works.

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